UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS (SYLLABUS D)

4024/02
Paper 2
October/November 2007
2 hours 30 minutes
Additional Materials: Answer Booklet/Paper
Graph paper (2 sheets)
Electronic calculator Geometrical instruments

Mathematical tables (optional)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

## Section A

Answer all questions.

## Section B

Answer any four questions.
Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

## Section A [52 marks]

Answer all the questions in this section.

1 (a)


From the top of a vertical tower, $A B$, an observer sees a car at $C$.
$A B=65 \mathrm{~m}$ and $C B=200 \mathrm{~m}$.
Calculate $C \hat{A} B$.
(b)


The diagram shows three positions at sea, $R, P$ and $S$.
$R$ is due north of $P$ and $S$ is due east of $P$.
$R P=300 \mathrm{~m}$ and $R S=750 \mathrm{~m}$.
(i) A boat sailed at a constant speed of $5 \mathrm{~km} / \mathrm{h}$ from $R$ to $S$.

It was at $R$ at 2256 .
Find the time it reached $S$.
(ii) Calculate the bearing of $S$ from $R$.

2 (a) It is given that $x=a+\sqrt{a^{2}+b^{2}}$.
(i) Calculate $x$ when $a=0.73$ and $b=1.84$.

Give your answer correct to 2 decimal places.
(ii) Express $b$ in terms of $x$ and $a$.
(b) A shopkeeper sells pens and pencils.

Each pen costs $\$ 5$ and each pencil costs $\$ 3$.
One day he sold $x$ pens.
On the same day he sold 9 more pens than pencils.
(i) Write down an expression, in terms of $x$, for his total income from the sale of these pens and pencils.
(ii) This total income was less than $\$ 300$.

Form an inequality in $x$ and solve it.
(iii) Hence write down the maximum number of pens that he sold.

3 (a)


In the diagram, $L M N$ is parallel to $P Q R$.
Angle $P Q M=124^{\circ}$ and $M Q=M R$.
Find
(i) $x$,
(ii) $y$.
(b) In a surveying exercise to find the distance between two points, $Y$ and $Z$, on opposite banks of a river, angles and lengths were measured.

$W X Z$ and $V X Y$ are straight lines.
$W \hat{V} X=Z \hat{Y} X=90^{\circ}$.
(i) Show that triangles $V W X$ and $Y Z X$ are similar.
(ii) $V W=25 \mathrm{~m}, V X=40 \mathrm{~m}$ and $X Y=160 \mathrm{~m}$.

Calculate the distance $Y Z$.

4 (a) In 2005, the cost of posting a letter was 28 cents.
A company posted 1200 letters and was given $4 \%$ discount on the cost.
Calculate the total discount.
(b) In 2006, the cost of posting a letter was increased from 28 cents to 35 cents.

Calculate the percentage increase in the cost of posting a letter.
(c) After the price increase to 35 cents, the cost to the company of posting 1200 letters was $\$ 399$.

Calculate the percentage discount that the company was given in 2006.
(d) In 2006, it cost $\$ 4.60$ to post a parcel.

This was an increase of $15 \%$ on the cost of posting the parcel in 2005.
Calculate the cost of posting this parcel in 2005.

5


The points $A, B, C$ and $D$ represent four towns on a map.
$A B C$ is a straight line.
$A B=24 \mathrm{~cm}, B D=16 \mathrm{~cm}$ and $C D=20 \mathrm{~cm}$.
Angle $A B D=112^{\circ}$.
(a) Calculate
(i) $A D$,
(ii) angle $B C D$,
(iii) the area of triangle $A B D$.
(b) The scale of the map is $1: 250000$.

Calculate the actual distance, in kilometres, from town $A$ to town $B$.

6 (a) Diagram I shows a design which consists of 7 congruent circles drawn inside a large circle. The circles touch at all the points shown.


Diagram I
(i) State the order of rotational symmetry of this design.
(ii) In Diagram II, two sections of the design have been shaded. Each small circle has a radius of 5 cm .


Diagram II
Calculate
(a) the area of the large circle,
(b) the shaded area.
(b) In Diagram III, circles, centres $O, A$ and $B$, are shown, together with the same shaded area. $O$ is also the centre of the large circle.


Diagram III
(i) Write down angle $A O B$.
(ii) Calculate the total perimeter of the shaded area.

## Section B [48 marks]

Answer four questions in this section.
Each question in this section carries 12 marks.

7 (a) Compost for growing plants consists of 3 parts of soil to 2 parts of sand to 1 part of peat.
(i) Calculate the number of litres of sand in a 75 litre bag of compost.
(ii) Compost is sold in 5 litre, 25 litre and 75 litre bags costing $\$ 2, \$ 8.75$ and $\$ 27$ respectively. Showing your working clearly, state which bag represents the best value for money.
(b) [The volume of a cone $=\frac{1}{3} \times$ base area $\times$ height.]

The diagram shows a plant pot.
The open end of the plant pot is a circle of radius 10 cm .
The closed end is a circle of radius 5 cm .
The height of the plant pot is 12 cm .
The plant pot is part of a right circular cone of height 24 cm .

(i) Calculate the volume of the plant pot. Give your answer in litres.
(ii) How many of these plant pots can be completely filled from a 75 litre bag of compost?
(iii) A smaller plant pot is geometrically similar to the original plant pot.

The open end of this plant pot is a circle of radius 5 cm .


How many of these plant pots can be completely filled from a 75 litre bag of compost?

## 8 Answer the whole of this question on a sheet of graph paper.

A stone was thrown from the top of a vertical cliff. Its position during the flight is represented by the equation $y=24+10 x-x^{2}$, where $y$ metres is the height of the stone above the sea and $x$ metres is the horizontal distance from the cliff.
(a) Solve the equation $0=24+10 x-x^{2}$.
(b) The table shows some values of $x$ and the corresponding values of $y$.

| $x$ | 0 | 2 | 4 | 6 | 8 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 24 | 40 | 48 | 48 | 40 | 24 |

(i) Using a scale of 1 cm to represent 1 metre, draw a horizontal $x$-axis for $0 \leqslant x \leqslant 14$.

Using a scale of 2 cm to represent 10 metres, draw a vertical $y$-axis for $0 \leqslant y \leqslant 50$.
On your axes, plot the points from the table and join them with a smooth curve.
(ii) Use your answer to part (a) to complete the graph which represents the flight of the stone. [1]
(iii) Find the height of the stone above the sea when its horizontal distance from the cliff was 7 m .
(iv) Use your graph to find how far the stone travelled horizontally while it was 6 m or more above the top of the cliff.
(c) It is given that $24+10 x-x^{2}=p-(x-5)^{2}$.
(i) Find the value of $p$.
(ii) Hence find
(a) the greatest height of the stone above the sea,
(b) the horizontal distance from the cliff when the stone was at its greatest height.

9 (a) Given that $\overrightarrow{P Q}=\binom{-2}{3}, \overrightarrow{Q R}=\binom{1}{0}$ and $\overrightarrow{R S}=\binom{1}{-5}$, find $\overrightarrow{P S}$.
(b)


In the diagram, $\overrightarrow{A B}=2 \mathbf{b}, \overrightarrow{A D}=3 \mathbf{a}$ and $\overrightarrow{D F}=\mathbf{b}-\mathbf{a}$.
$E$ is the midpoint of $A B$ and $F$ is the midpoint of $D C$.
(i) Express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,
(a) $\overrightarrow{E A}$,
(b) $\overrightarrow{D C}$,
(c) $\overrightarrow{E F}$,
(d) $\overrightarrow{B C}$.
(ii) (a) Give the special name of the quadrilateral $A B C D$.

Give your reason.
(b) Find the ratio $|\overrightarrow{B C}|:|\overrightarrow{E F}|:|\overrightarrow{A D}|$.
(c)


Points $A, B, C$ and $D$ lie on Circle I.
$O$ is the centre of Circle I.
Points $A, O, E, C$ and $F$ lie on Circle II.
$A E B$ and $A D F$ are straight lines.
Angle $A O C=146^{\circ}$.
Giving your reasons, write down
(i) $C \hat{E} A$,
(ii) $C \hat{B} A$,
(iii) $C \hat{F} A$,
(iv) $D \hat{C} F$.

## 10 Answer the whole of this question on a sheet of graph paper.

Potatoes are sold in sacks.
One sack, picked at random, contained 260 potatoes.
The masses, in grams, of the potatoes in this sack are summarised in the table below.

| Mass <br> $(m$ grams $)$ | $50<m \leqslant 100$ | $100<m \leqslant 150$ | $150<m \leqslant 200$ | $200<m \leqslant 250$ | $250<m \leqslant 300$ | $300<m \leqslant 350$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 4 | 56 | 84 | 76 | 36 | 4 |

(a) (i) Calculate an estimate of the total mass of the potatoes in this sack.
(ii) Calculate an estimate of the mean mass, in grams, of a potato.
(b) (i) Copy and complete the cumulative frequency table given below.

| Mass <br> $(m$ grams $)$ | $m \leqslant 50$ | $m \leqslant 100$ | $m \leqslant 150$ | $m \leqslant 200$ | $m \leqslant 250$ | $m \leqslant 300$ | $m \leqslant 350$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cumulative <br> frequency | 0 | 4 | 60 |  |  |  | 260 |

(ii) Using a scale of 2 cm to represent 50 grams, draw a horizontal axis for masses between 0 and 350 grams.
Using a scale of 2 cm to represent 50 potatoes, draw a vertical axis for values from 0 to 300 .
On your axes, draw a smooth cumulative frequency curve to illustrate this information.
(iii) Use your curve to find
(a) the median,
(b) the interquartile range.
(c) The organisers of a barbecue expect to sell 500 baked potatoes.

Each potato should have a mass greater than 200 g .
Estimate the number of sacks of potatoes they will need.

11 (a) $P$ is the point $(2,9)$ and $Q$ is the point $(4,6)$.
Find
(i) the length of $P Q$,
(ii) the equation of the line $P Q$.
(b)


The diagram shows triangles $A, B, C$ and $D$.
(i) Find the matrix representing the transformation that maps triangle $A$ onto triangle $B$.
(ii) Describe fully the single transformation that maps triangle $B$ onto triangle $C$.
(iii) Triangle $C$ is mapped onto triangle $D$ by the translation T .
(a) Write down the column vector that represents T .
(b) The transformation R that maps triangle $A$ onto triangle $C$ is
represented by the matrix $\left(\begin{array}{rr}0 & -1 \\ 1 & 0\end{array}\right)$.
Show that the transformation R followed by T maps $(h, k)$ onto $(-k-3, h-3)$.
(c) Find the value of $h$ and the value of $k$ for which the transformation R followed by T maps ( $h, k$ ) onto itself.
(d) The single transformation that is equivalent to R followed by T is a rotation.

Write down the coordinates of the centre of this rotation.

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